# Study on Bezier curve discrete two-dimensional shear beam method based on absolute nodal coordinate formulation

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## EXTENDED ABSTRACT

The finite element method is a discretization method widely used in flexible multibody system dynamics. However, when the actual shape of the flexible body is irregular, in order to obtain sufficient accuracy, the number of system dynamics equations obtained by the finite element method is often too large to have high computational efficiency. Therefore, the discretization method suitable for geometric nonlinear problems has important research significance. Bézier and B-spline are widely used to describe complex geometric models in CAD systems. At present, Bézier curve can not only be applied to dynamic analysis of rotating flexible beam, but also have been extended to plate and solid element [1]. On the basis that Bézier curves can only discretize one-dimensional beam elements [2], a discretization method suitable for treating two-dimensional shear beams is proposed.

Based on Timoshenko beam theory, Omar and Shabana proposed a two-dimensional shear deformation beam element [3], but cubic Bézier curves with four control points cannot describe the shear deformation of two-dimensional beam elements. Therefore, on the premise of not changing the excellent properties of Bézier curves, this paper proposes to add new control points above the control points at both ends of Bézier curves. The shear deformation of the beam cross-section can be described by the new control points and the position coordinates of curve endpoints. The relationships of Bézier curve control points, slope and position coordinates are shown in Fig. 1.

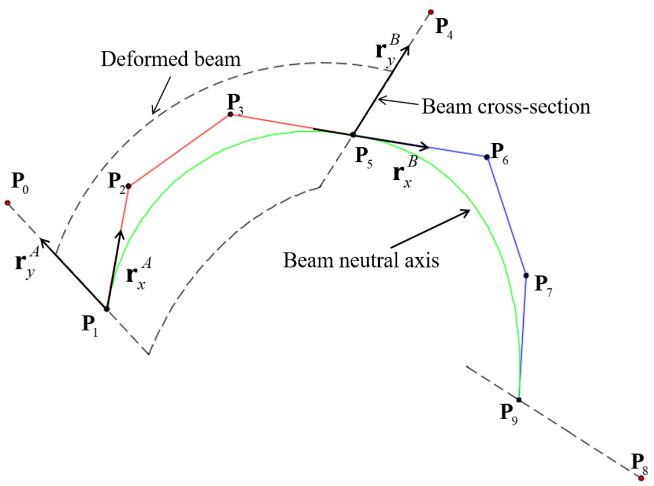


Figure 1: The relationships of Bézier curve control points, slope and position coordinates

The transformation relationship between Bézier curve control points and ANCF (absolute nodal coordinate formulation) element node coordinates and slope is given as

$$
\begin{aligned}
&\mathbf{r}^A = \mathbf{r}(0), && \mathbf{r}^B = \mathbf{r}(l), \\
&\mathbf{r}_x^A = \left.\frac{\partial \mathbf{r}(t,s)}{\partial x}\right|_{x=0,y=0} = \left.\frac{\partial \mathbf{r}(t,s)}{\partial t}\frac{\partial t}{\partial x}\right|_{t=0,s=0}, && \mathbf{r}_x^B = \left.\frac{\partial \mathbf{r}(t,s)}{\partial x}\right|_{x=l,y=0} = \left.\frac{\partial \mathbf{r}(t,s)}{\partial t}\frac{\partial t}{\partial x}\right|_{t=1,s=0}, \\
&\mathbf{r}_y^A = \left.\frac{\partial \mathbf{r}(t,s)}{\partial y}\right|_{x=0} = \left.\frac{\partial \mathbf{r}(t,s)}{\partial t}\frac{\partial t}{\partial y}\right|_{t=0}, && \mathbf{r}_y^B = \left.\frac{\partial \mathbf{r}(t,s)}{\partial y}\right|_{x=l} = \left.\frac{\partial \mathbf{r}(t,s)}{\partial t}\frac{\partial t}{\partial y}\right|_{t=1}
\end{aligned}
\tag{1}
$$

where $t = x/l$, $\partial t/\partial x = 1/l$, $s = y/l$, $\partial s/\partial y = 1/l$, and $l$ is the length of the cell. According to the properties of Bézier curves at the endpoints, the position function of the two dimensional shear beam element is obtained

$$\begin{aligned}\mathbf{r} &= (1-t)^3\mathbf{P}_1 + 3t(1-t)^2\mathbf{P}_2 + 3t^2(1-t)\mathbf{P}_3 + t^3\mathbf{P}_5 + s(1-t)(\mathbf{P}_0-\mathbf{P}_1) + st(\mathbf{P}_4-\mathbf{P}_5)\\ &= s(1-t)\mathbf{P}_0 + \left[(1-t)^3 - s(1-t)\right]\mathbf{P}_1 + 3t(1-t)^2\mathbf{P}_2 + 3t^2(1-t)\mathbf{P}_3 + st\mathbf{P}_4 + \left(t^3-st\right)\mathbf{P}_5\end{aligned} \tag{2}$$

According to Eq. (2), this method replaces the position vector and gradient vector of nodes in ANCF element with the position vector of spline function control point, realizing automatic transformation between the two elements, which is also very necessary for the successful integration of computer aided design and analysis (I-CAD-A) [4].

Finally, a flexible pendulum in free fall under the action of gravity is taken as an example. By comparing the time-varying displacement curves of the endpoints of the flexible beam calculated by the two elements, the accuracy and convergence of the new element are verified.

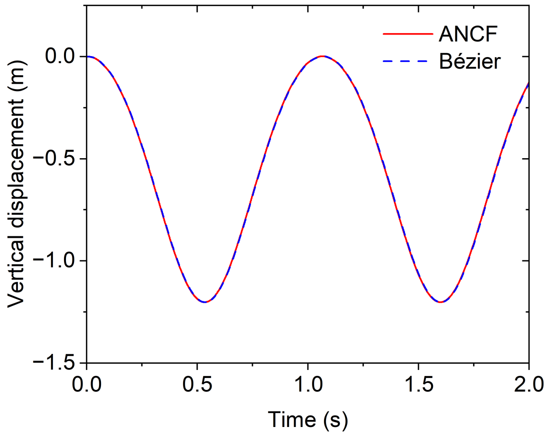


Figure 2: Displacement of the beam tip point

According to the images of the vertical displacement of the tip of the flexible pendulum changing with time, the improved Bézier curve is consistent with the results obtained by the ANCF 2D shear beam element, which indicates that the new element constructed based on the improved Bézier curve is reasonable to some extent. The preliminary conclusions of this study are as follows:

1. When the number of units is the same, the new element has faster computational efficiency than the ANCF method.

2. The new element inherits the excellent properties of Bézier curves and has a broad prospect in dealing with geometric nonlinear models compared with the traditional finite element method.

3. The new element solves the Bézier curve discrete two-dimensional shear beam problem.

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